Robotics I - Homework 1

Direct kinematics of the Comau Smart 5 NJ4 170 robot

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1 Robot frames and table of Denavit-Hartenberg parameters

The following views should help to understand the reference frames assigned to the robot.

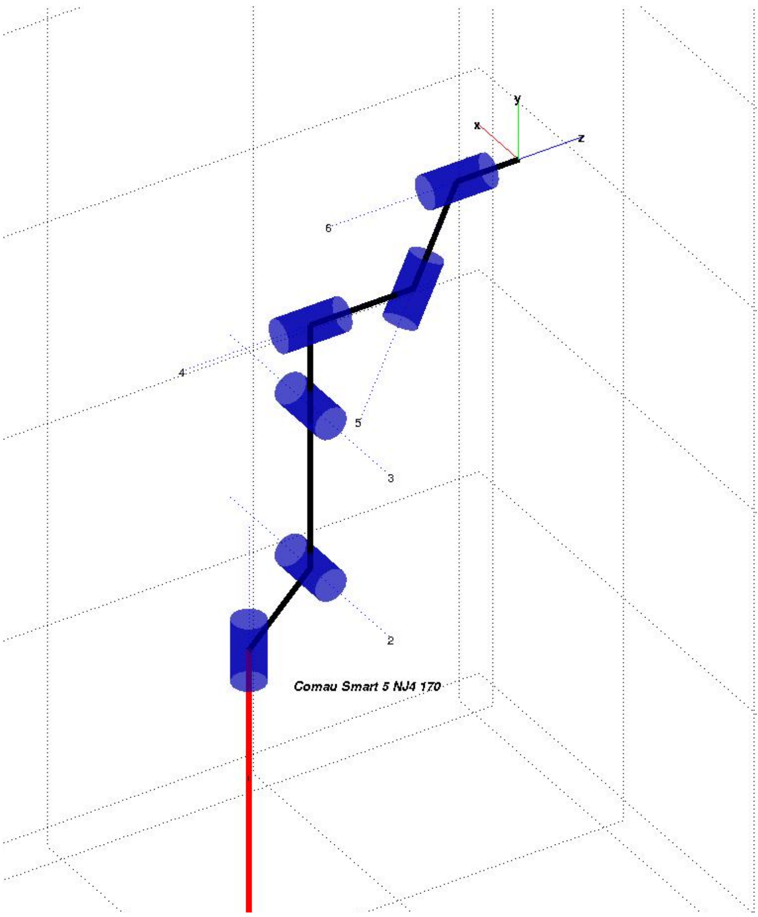

Figure 1: First view of the kinematic structure

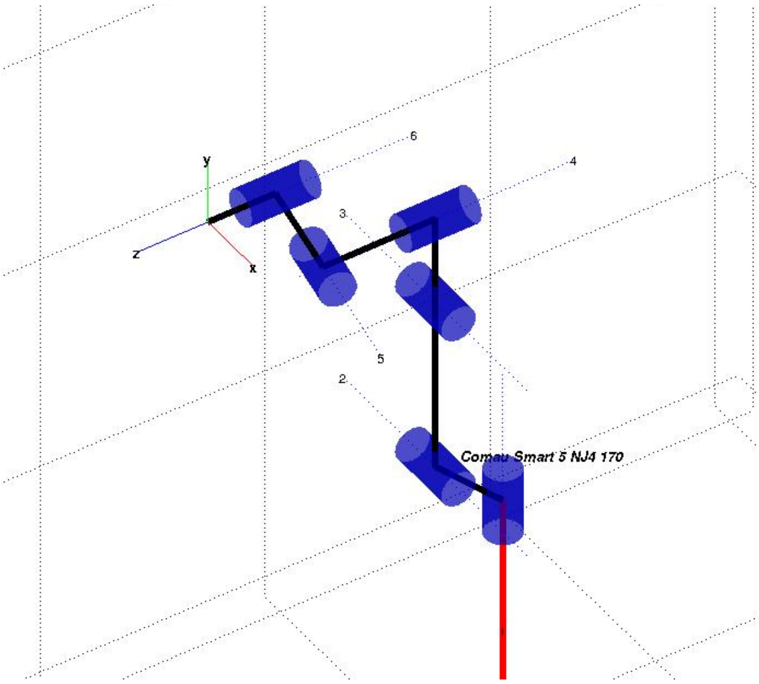

Figure 2: Second view of the kinematic structure

Here is the table of Denavit-Hartenberg parameters. Symbols are used for numerically unknown quantities.

Joint i	θ_i	d_i	α_i	a_i
1	0	d_i	$-\frac{\pi}{2}$	a_1
2	$-\frac{\pi}{2}$	0	0	a_2
3	0	0	$-\frac{\pi}{2}$	a_3
4	$\frac{\pi}{2}$	d_4	$65\circ$	0
5	$-\pi$	d_5	$65\circ$	0
6	0	d_6	0	0

Table 1: Denavit-Hartenberg: convention associated table of parameters

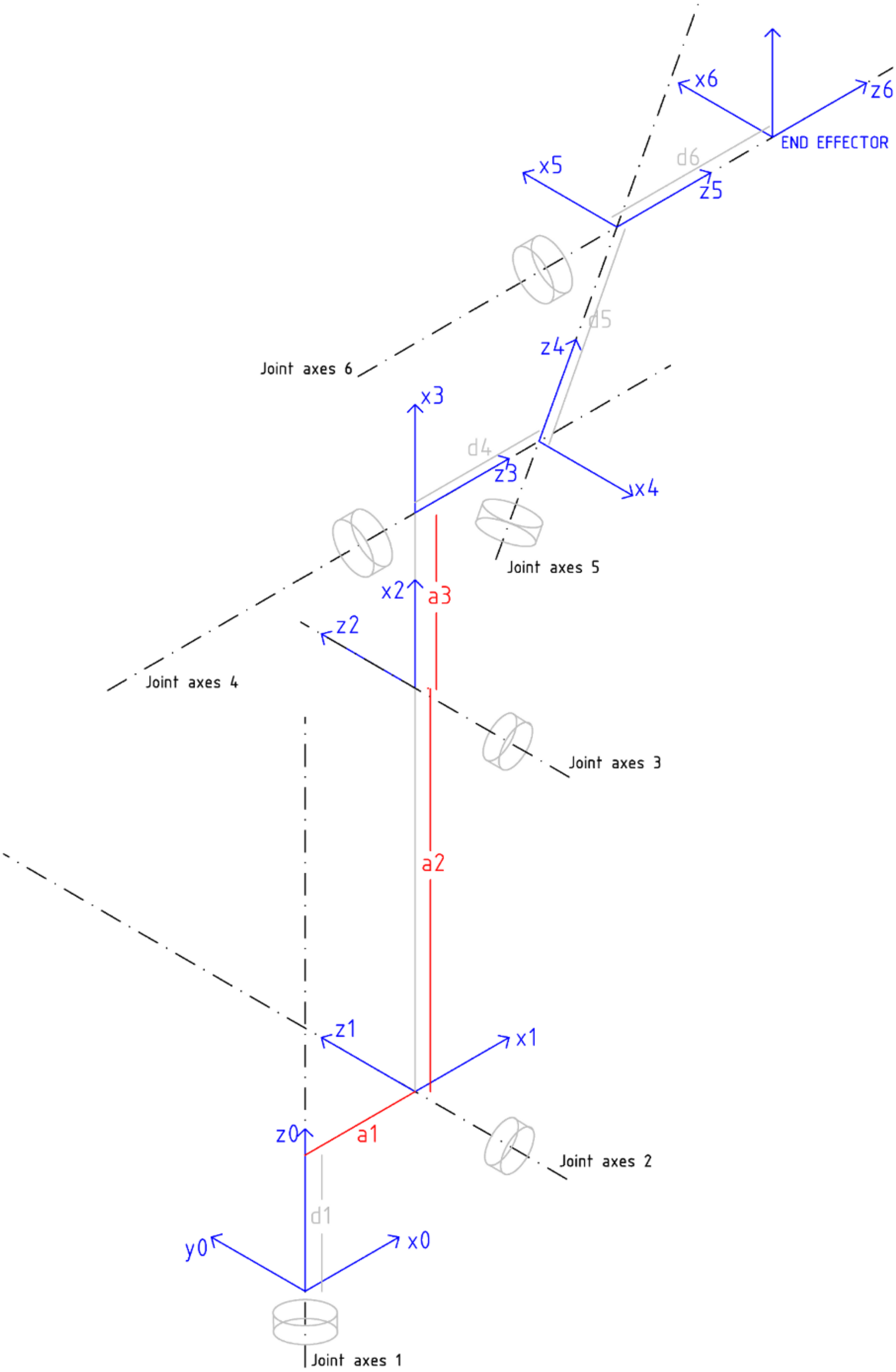

Figure 3: Reference frames assigned to the robot

2 Direct kinematics

The matlab function shown above was written at the beginning in order to simplify notation in the final matlab script that computes the direct kinematics.

```
function out1 = homogeneous_transformation(theta,d,alfa,a)

out1 = [cos(theta)  -cos(alfa)*sin(theta)    sin(alfa)*sin(theta)   a*cos(theta)   ;
        sin(theta)   cos(alfa)*cos(theta)   -sin(alfa)*cos(theta)   a*sin(theta)   ;
        0            sin(alfa)               cos(alfa)              d              ;
        0            0                       0                      1             ];
```

Then the direct kinematics of the Comau robot was computed by the following matlab script.

```
% Solution of Robotics I - Homework 1
% Direct kinematics of Comau Smart5 NJ4 170 robot
%
% by Paolo Forni, November 20, 2011

% Declaration of symbolic variables I need
syms a1 a2 a3 d1 d4 d5 d6 q1 q2 q3 q4 q5 q6

% Computation of each homogeneous transformation
disp('Homogeneous transformation from frame 0 to frame 1');
A01 = homogeneous_transformation(q1,d1,sym('-pi/2'),a1)

disp('Homogeneous transformation from frame 1 to frame 2');
A12 = homogeneous_transformation(q2,sym('0'),sym('0'),a2)

disp('Homogeneous transformation from frame 2 to frame 3');
A23 = homogeneous_transformation(q3,sym('0'),sym('-pi/2'),a3)

disp('Homogeneous transformation from frame 3 to frame 4');
A34 = homogeneous_transformation(q4,d4,sym('65/180*pi'),sym('0'))

disp('Homogeneous transformation from frame 4 to frame 5');
A45 = homogeneous_transformation(q5,d5,sym('65/180*pi'),sym('0'))

disp('Homogeneous transformation from frame 5 to frame 6');
A56 = homogeneous_transformation(q6,d6,sym('0'),sym('0'))

% I compute some intermediate transformation in order to verify matlab with
% my computation
A03raw = A01*A12*A23;

disp('_____________________________________________________')
disp('Intermediate calculation: homogeneous transformation from frame 0 to frame 3');
A03 = simplify(A03raw);

n03 = A03(1:3,1)
s03 = A03(1:3,2)
a03 = A03(1:3,3)
p03 = A03(1:3,4)

A36raw = A34*A45*A56;

disp('_____________________________________________________')
disp('Intermediate calculation: homogeneous transformation from frame 3 to frame 6');
A36 = simplify(A36raw);
```

```
n36 = A36(1:3,1)
s36 = A36(1:3,2)
a36 = A36(1:3,3)
p36 = A36(1:3,4)

A06raw = A03*A36;

% Final DIRECT KINEMATICS
disp('_____________________________________________________')
disp('Final direct kinematics of COMAU Smart5 NJ4 170 robot')
A06 = simplify(A06raw);
n06 = A06(1:3,1)
s06 = A06(1:3,2)
a06 = A06(1:3,3)
p06 = A06(1:3,4)
```

The output of the matlab script is the following:

```
Homogeneous transformation from frame 0 to frame 1

A01 =

[ cos(q1),  0, -sin(q1), a1*cos(q1)]
[ sin(q1),  0,  cos(q1), a1*sin(q1)]
[       0, -1,        0,         d1]
[       0,  0,        0,          1]

Homogeneous transformation from frame 1 to frame 2

A12 =

[ cos(q2), -sin(q2), 0, a2*cos(q2)]
[ sin(q2),  cos(q2), 0, a2*sin(q2)]
[       0,        0, 1,          0]
[       0,        0, 0,          1]

Homogeneous transformation from frame 2 to frame 3

A23 =

[ cos(q3),  0, -sin(q3), a3*cos(q3)]
[ sin(q3),  0,  cos(q3), a3*sin(q3)]
[       0, -1,        0,          0]
[       0,  0,        0,          1]

Homogeneous transformation from frame 3 to frame 4

A34 =

[ cos(q4), -cos((13*pi)/36)*sin(q4),  sin((13*pi)/36)*sin(q4),  0]
[ sin(q4),  cos((13*pi)/36)*cos(q4), -sin((13*pi)/36)*cos(q4),  0]
[       0,          sin((13*pi)/36),          cos((13*pi)/36), d4]
[       0,                        0,                        0,  1]

Homogeneous transformation from frame 4 to frame 5

A45 =

[ cos(q5), -cos((13*pi)/36)*sin(q5),  sin((13*pi)/36)*sin(q5),  0]
```

```
[ sin(q5),  cos((13*pi)/36)*cos(q5), -sin((13*pi)/36)*cos(q5),  0]
[       0,          sin((13*pi)/36),          cos((13*pi)/36), d5]
[       0,                        0,                        0,  1]
```

Homogeneous transformation from frame 5 to frame 6

```
A56 =

[ cos(q6), -sin(q6), 0,  0]
[ sin(q6),  cos(q6), 0,  0]
[       0,        0, 1, d6]
[       0,        0, 0,  1]
```

Intermediate calculation: homogeneous transformation from frame 0 to frame 3

```
n03 =

 cos(q2 + q3)*cos(q1)
 cos(q2 + q3)*sin(q1)
        -sin(q2 + q3)


s03 =

  sin(q1)
 -cos(q1)
        0


a03 =

 -sin(q2 + q3)*cos(q1)
 -sin(q2 + q3)*sin(q1)
         -cos(q2 + q3)


p03 =

 cos(q1)*(a1 + a3*cos(q2 + q3) + a2*cos(q2))
 sin(q1)*(a1 + a3*cos(q2 + q3) + a2*cos(q2))
           d1 - a3*sin(q2 + q3) - a2*sin(q2)
```

Intermediate calculation: homogeneous transformation from frame 3 to frame 6

```
n36 =

  cos(q6)*(cos(q4)*cos(q5) - cos((13*pi)/36)*sin(q4)*sin(q5)) +
 -sin(q6)*(sin(q4)*(cos((13*pi)/36)^2 - 1) +
 +cos((13*pi)/36)*cos(q4)*sin(q5) + cos((13*pi)/36)^2*cos(q5)*sin(q4))

  sin(q6)*(cos(q4)*(cos((13*pi)/36)^2 - 1) - cos((13*pi)/36)*sin(q4)*sin(q5) +
 +cos((13*pi)/36)^2*cos(q4)*cos(q5)) + cos(q6)*(cos(q5)*sin(q4) +
 +cos((13*pi)/36)*cos(q4)*sin(q5))

  sin(q6)*(cos((13*pi)/36)*sin((13*pi)/36) +
 +cos((13*pi)/36)*sin((13*pi)/36)*cos(q5)) + sin((13*pi)/36)*cos(q6)*sin(q5)
```

```
s36 =

 -cos(q6)*(sin(q4)*(cos((13*pi)/36)^2 - 1) + cos((13*pi)/36)*cos(q4)*sin(q5) +
 +cos((13*pi)/36)^2*cos(q5)*sin(q4)) - sin(q6)*(cos(q4)*cos(q5) +
 -cos((13*pi)/36)*sin(q4)*sin(q5))

  cos(q6)*(cos(q4)*(cos((13*pi)/36)^2 - 1) - cos((13*pi)/36)*sin(q4)*sin(q5) +
 +cos((13*pi)/36)^2*cos(q4)*cos(q5)) - sin(q6)*(cos(q5)*sin(q4) +
 +cos((13*pi)/36)*cos(q4)*sin(q5))

  cos(q6)*(cos((13*pi)/36)*sin((13*pi)/36) + cos((13*pi)/36)*sin((13*pi)/36)*cos(q5)) +
 -sin((13*pi)/36)*sin(q5)*sin(q6)


a36 =

  sin((13*pi)/36)*(cos(q4)*sin(q5) + cos((13*pi)/36)*sin(q4) +
 +cos((13*pi)/36)*cos(q5)*sin(q4))

 -sin((13*pi)/36)*(cos((13*pi)/36)*cos(q4) - sin(q4)*sin(q5) +
 +cos((13*pi)/36)*cos(q4)*cos(q5))

 1 - 2*cos((5*pi)/36)^2*cos(q5/2)^2


p36 =

  sin((13*pi)/36)*(d5*sin(q4) + d6*cos(q4)*sin(q5) +
 +d6*cos((13*pi)/36)*sin(q4) + d6*cos((13*pi)/36)*cos(q5)*sin(q4))

 -sin((13*pi)/36)*(d5*cos(q4) - d6*sin(q4)*sin(q5) +
 +d6*cos((13*pi)/36)*cos(q4) + d6*cos((13*pi)/36)*cos(q4)*cos(q5))

  d4 + d6*(cos(q5)*(cos((13*pi)/36)^2 - 1) + cos((13*pi)/36)^2) +
 +d5*cos((13*pi)/36)

-------------------------------------------------------
Final direct kinematics of COMAU Smart5 NJ4 170 robot

n06 =

  sin(q1)*(sin(q6)*(cos(q4)*(cos((13*pi)/36)^2 - 1) - cos((13*pi)/36)*sin(q4)*sin(q5) +
 +cos((13*pi)/36)^2*cos(q4)*cos(q5)) + cos(q6)*(cos(q5)*sin(q4) +
 +cos((13*pi)/36)*cos(q4)*sin(q5))) +
 -sin(q2 + q3)*cos(q1)*(sin(q6)*(cos((13*pi)/36)*sin((13*pi)/36) +
 +cos((13*pi)/36)*sin((13*pi)/36)*cos(q5)) + sin((13*pi)/36)*cos(q6)*sin(q5)) +
 -cos(q2 + q3)*cos(q1)*(sin(q6)*(sin(q4)*(cos((13*pi)/36)^2 - 1) +
 +cos((13*pi)/36)*cos(q4)*sin(q5) + cos((13*pi)/36)^2*cos(q5)*sin(q4)) +
 -cos(q6)*(cos(q4)*cos(q5) - cos((13*pi)/36)*sin(q4)*sin(q5)))

 -cos(q1)*(sin(q6)*(cos(q4)*(cos((13*pi)/36)^2 - 1) - cos((13*pi)/36)*sin(q4)*sin(q5) +
 +cos((13*pi)/36)^2*cos(q4)*cos(q5)) + cos(q6)*(cos(q5)*sin(q4) +
 +cos((13*pi)/36)*cos(q4)*sin(q5))) +
 -sin(q2 + q3)*sin(q1)*(sin(q6)*(cos((13*pi)/36)*sin((13*pi)/36) +
 +cos((13*pi)/36)*sin((13*pi)/36)*cos(q5)) + sin((13*pi)/36)*cos(q6)*sin(q5)) +
 -cos(q2 + q3)*sin(q1)*(sin(q6)*(sin(q4)*(cos((13*pi)/36)^2 - 1) +
```

```
+cos((13*pi)/36)*cos(q4)*sin(q5) + cos((13*pi)/36)^2*cos(q5)*sin(q4)) +
-cos(q6)*(cos(q4)*cos(q5) - cos((13*pi)/36)*sin(q4)*sin(q5)))

 sin(q2 + q3)*(sin(q6)*(sin(q4)*(cos((13*pi)/36)^2 - 1) +
+cos((13*pi)/36)*cos(q4)*sin(q5) + cos((13*pi)/36)^2*cos(q5)*sin(q4)) +
-cos(q6)*(cos(q4)*cos(q5) - cos((13*pi)/36)*sin(q4)*sin(q5))) +
-cos(q2 + q3)*(sin(q6)*(cos((13*pi)/36)*sin((13*pi)/36) +
 cos((13*pi)/36)*sin((13*pi)/36)*cos(q5)) + sin((13*pi)/36)*cos(q6)*sin(q5))


s06 =

 sin(q1)*(cos(q6)*(cos(q4)*(cos((13*pi)/36)^2 - 1) - cos((13*pi)/36)*sin(q4)*sin(q5) +
+cos((13*pi)/36)^2*cos(q4)*cos(q5)) - sin(q6)*(cos(q5)*sin(q4) +
+cos((13*pi)/36)*cos(q4)*sin(q5))) +
-sin(q2 + q3)*cos(q1)*(cos(q6)*(cos((13*pi)/36)*sin((13*pi)/36) +
+cos((13*pi)/36)*sin((13*pi)/36)*cos(q5)) - sin((13*pi)/36)*sin(q5)*sin(q6))
-cos(q2 + q3)*cos(q1)*(cos(q6)*(sin(q4)*(cos((13*pi)/36)^2 - 1) +
+cos((13*pi)/36)*cos(q4)*sin(q5) + cos((13*pi)/36)^2*cos(q5)*sin(q4)) +
+sin(q6)*(cos(q4)*cos(q5) - cos((13*pi)/36)*sin(q4)*sin(q5)))

-cos(q1)*(cos(q6)*(cos(q4)*(cos((13*pi)/36)^2 - 1) - cos((13*pi)/36)*sin(q4)*sin(q5) +
+cos((13*pi)/36)^2*cos(q4)*cos(q5)) - sin(q6)*(cos(q5)*sin(q4) +
+cos((13*pi)/36)*cos(q4)*sin(q5))) +
-sin(q2 + q3)*sin(q1)*(cos(q6)*(cos((13*pi)/36)*sin((13*pi)/36) +
+cos((13*pi)/36)*sin((13*pi)/36)*cos(q5)) - sin((13*pi)/36)*sin(q5)*sin(q6)) +
-cos(q2 + q3)*sin(q1)*(cos(q6)*(sin(q4)*(cos((13*pi)/36)^2 - 1) +
+cos((13*pi)/36)*cos(q4)*sin(q5) + cos((13*pi)/36)^2*cos(q5)*sin(q4)) +
+ sin(q6)*(cos(q4)*cos(q5) - cos((13*pi)/36)*sin(q4)*sin(q5)))

 sin(q2 + q3)*(cos(q6)*(sin(q4)*(cos((13*pi)/36)^2 - 1) +
+cos((13*pi)/36)*cos(q4)*sin(q5) +
+cos((13*pi)/36)^2*cos(q5)*sin(q4)) + sin(q6)*(cos(q4)*cos(q5) +
-cos((13*pi)/36)*sin(q4)*sin(q5))) +
-cos(q2 + q3)*(cos(q6)*(cos((13*pi)/36)*sin((13*pi)/36) +
 cos((13*pi)/36)*sin((13*pi)/36)*cos(q5)) - sin((13*pi)/36)*sin(q5)*sin(q6))


a06 =

sin(q2 + q3)*cos(q1)*(2*cos((5*pi)/36)^2*cos(q5/2)^2 - 1) +
-sin((13*pi)/36)*sin(q1)*(cos((13*pi)/36)*cos(q4) - sin(q4)*sin(q5) +
+cos((13*pi)/36)*cos(q4)*cos(q5)) +
+sin((13*pi)/36)*cos(q2 + q3)*cos(q1)*(cos(q4)*sin(q5) +
+cos((13*pi)/36)*sin(q4) + cos((13*pi)/36)*cos(q5)*sin(q4))

 sin((13*pi)/36)*cos(q1)*(cos((13*pi)/36)*cos(q4) - sin(q4)*sin(q5) +
+cos((13*pi)/36)*cos(q4)*cos(q5)) +
+sin(q2 + q3)*sin(q1)*(2*cos((5*pi)/36)^2*cos(q5/2)^2 - 1)
+sin((13*pi)/36)*cos(q2 + q3)*sin(q1)*(cos(q4)*sin(q5) + cos((13*pi)/36)*sin(q4) +
cos((13*pi)/36)*cos(q5)*sin(q4))

 cos(q2 + q3)*(2*cos((5*pi)/36)^2*cos(q5/2)^2 - 1) +
-sin((13*pi)/36)*sin(q2 + q3)*(cos(q4)*sin(q5) + cos((13*pi)/36)*sin(q4) +
+cos((13*pi)/36)*cos(q5)*sin(q4))


p06 =
```

```
cos(q1)*(a1 + a3*cos(q2 + q3) + a2*cos(q2)) - sin((13*pi)/36)*sin(q1)*(d5*cos(q4) +
-d6*sin(q4)*sin(q5) + d6*cos((13*pi)/36)*cos(q4) + d6*cos((13*pi)/36)*cos(q4)*cos(q5)) +
-sin(q2 + q3)*cos(q1)*(d4 + d6*(cos(q5)*(cos((13*pi)/36)^2 - 1) + cos((13*pi)/36)^2) +
+d5*cos((13*pi)/36)) + sin((13*pi)/36)*cos(q2 + q3)*cos(q1)*(d5*sin(q4) +
+d6*cos(q4)*sin(q5) + d6*cos((13*pi)/36)*sin(q4) + d6*cos((13*pi)/36)*cos(q5)*sin(q4))

 sin(q1)*(a1 + a3*cos(q2 + q3) + a2*cos(q2)) + sin((13*pi)/36)*cos(q1)*(d5*cos(q4) +
-d6*sin(q4)*sin(q5) + d6*cos((13*pi)/36)*cos(q4) + d6*cos((13*pi)/36)*cos(q4)*cos(q5)) +
-sin(q2 + q3)*sin(q1)*(d4 + d6*(cos(q5)*(cos((13*pi)/36)^2 - 1) + cos((13*pi)/36)^2) +
+d5*cos((13*pi)/36)) + sin((13*pi)/36)*cos(q2 + q3)*sin(q1)*(d5*sin(q4) +
+d6*cos(q4)*sin(q5) + d6*cos((13*pi)/36)*sin(q4) +
+d6*cos((13*pi)/36)*cos(q5)*sin(q4))

 d1 - cos(q2 + q3)*(d4 + d6*(cos(q5)*(cos((13*pi)/36)^2 - 1) + cos((13*pi)/36)^2) +
+d5*cos((13*pi)/36)) - a3*sin(q2 + q3) - a2*sin(q2) +
-sin((13*pi)/36)*sin(q2 + q3)*(d5*sin(q4) + d6*cos(q4)*sin(q5) +
+d6*cos((13*pi)/36)*sin(q4) + d6*cos((13*pi)/36)*cos(q5)*sin(q4))
```

3 Zero configuration

The following views should help to understand the zero configuration of the robot.

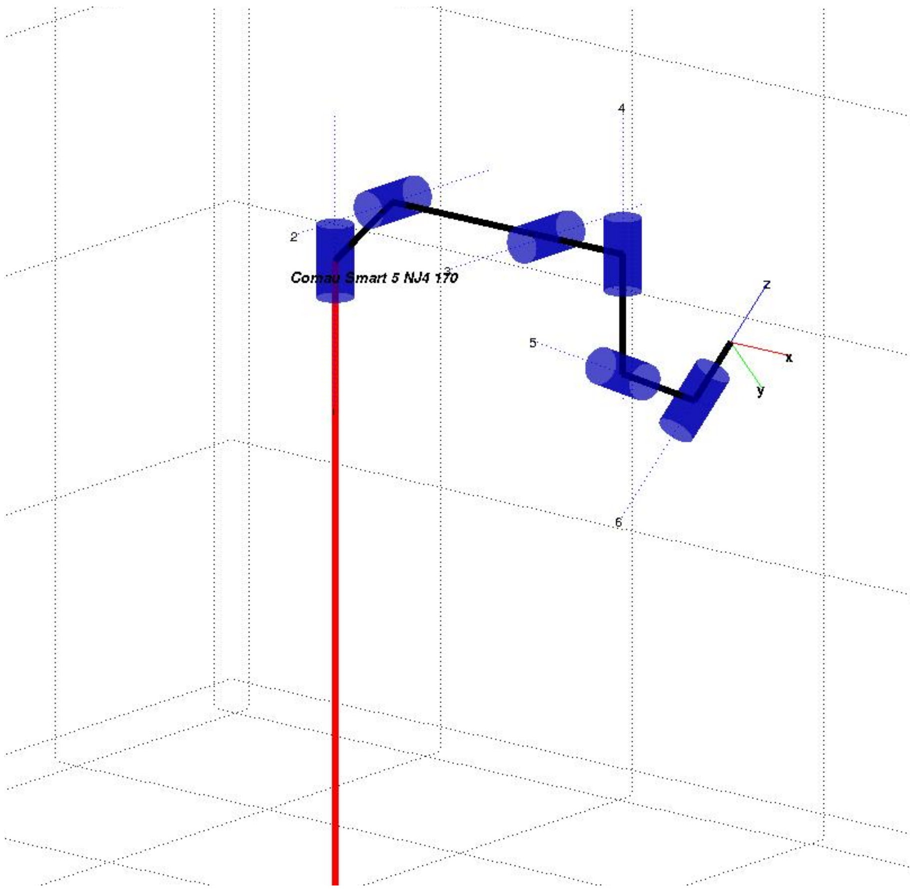

Figure 4: First view of zero configuration

Finally there is another matlab script that gives the expression of the position of origin O_6 of the end effector and of the position of intersection point H between joint axes 4 and 5.

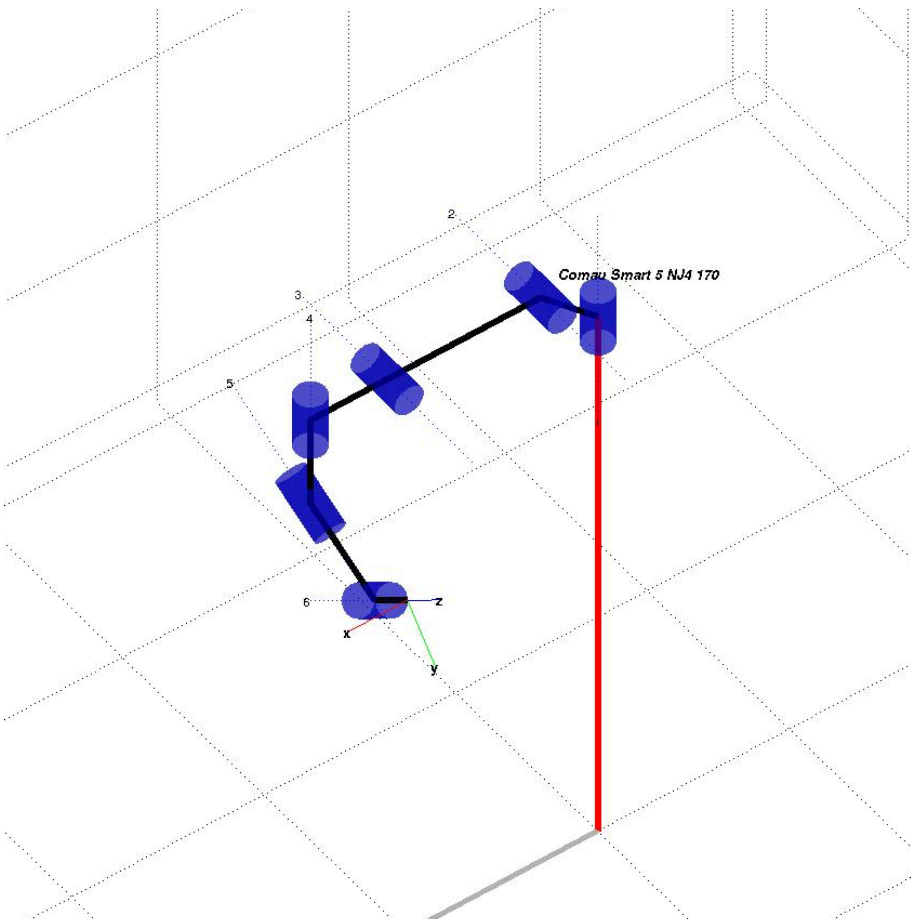

Figure 5: Second view of zero configuration

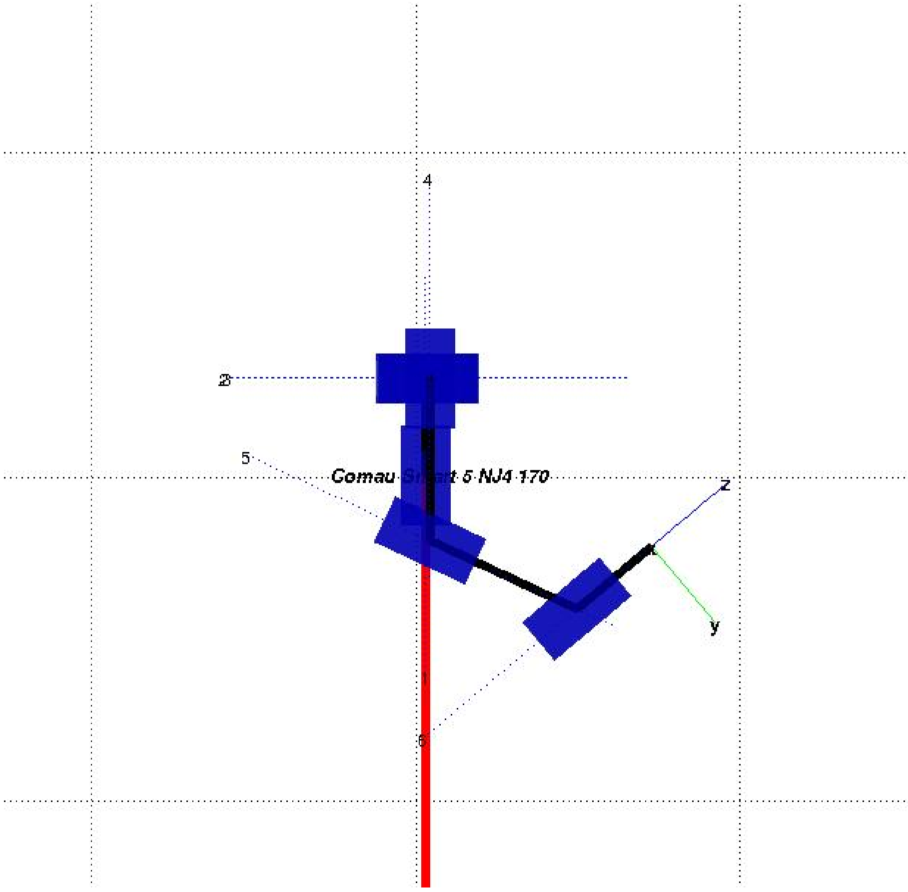

Figure 6: Third view of zero configuration

```
% Solution of Robotics I - Homework 1
% Direct kinematics of Comau Smart5 NJ4 170 robot
% AT ZERO CONFIGURATION
%
% by Paolo Forni, November 20, 2011

% Declaration of symbolic variables I need
syms a1 a2 a3 d1 d4 d5 d6 q1 q2 q3 q4 q5 q6

% Computation of each homogeneous transformation
A01 = homogeneous_transformation(sym('0'),d1,sym('-pi/2'),a1);
A12 = homogeneous_transformation(sym('0'),sym('0'),sym('0'),a2);
A23 = homogeneous_transformation(sym('0'),sym('0'),sym('-pi/2'),a3);
A34 = homogeneous_transformation(sym('0'),d4,sym('65/180*pi'),sym('0'));
A45 = homogeneous_transformation(sym('0'),d5,sym('65/180*pi'),sym('0'));
A56 = homogeneous_transformation(sym('0'),d6,sym('0'),sym('0'));

%
% Computation
%

A04 = A01*A12*A23*A34;
A04 = simplify(A04);
disp('_____________________________________________________')
disp('Position of intersection point H between joint axes 4 and 5:')
p04 = A04(1:3,4)

A06 = A04*A45*A56;
A06 = simplify(A06);

disp('_____________________________________________________')
disp('Position of the origin O6 of the end-effector frame:')
p06 = A06(1:3,4)
```

And this is the output:

```
_____________________________________________________
Position of intersection point H between joint axes 4 and 5:

p04 =

 a1 + a2 + a3
            0
      d1 - d4

_____________________________________________________
Position of the origin O6 of the end-effector frame:

p06 =

                                                   a1 + a2 + a3
                        d5*sin((13*pi)/36) + d6*sin((5*pi)/18)
 - 2*d6*cos((13*pi)/36)^2 - d5*cos((13*pi)/36) + d1 - d4 + d6
```

List of Figures